Fractal Analysis of Hyperbolic Saddles and Semi-Hyperbolic Singularities with Applications

Vlatko Crnković

University of Zagreb
and
Hasselt University

It is already well known that the multiplicity/cyclicity of limit cycles and weak foci of analytic planar vector fields can be determined from the Minkowski dimension of spiral trajectories near such limit periodic sets(see [5], [6]). In these configurations the intersection of a spiral trjectory with any transversal to the limit cycle/weak focus has the same Minkowski dimension. On the other hand, non-regular limit periodic sets such as saddle-loops and polycycles don't necessarily satisfy this property which is one of the obstacles in finding the link between the Minkowski dimension of spiral trajectories and that of the intersections of the trajectory with transversals to the limit periodic set. The main difficulty arises near singular points on limit periodic set. The dimension of the intersection of the spiral trajectory with a transversal changes as we move the transversal through a singular point. We present two new results that deal with these difficulties and allow us to express the dimension of the parts of spiral trajectories near two types of singularities: hyperbolic saddles and semi-hyperbolic singularities with saddle-like behaviour. We also prese the application of these results to two examples of non-regular limit periodic sets: the saddle-loop and the hyperbolic 2-cycle. Based on previous well established theory ([4], [2], [1]), we provide a way to obtain upper bounds on the cyclicity of such objects using the Minkowski dimension of (any) spiral trajectories(y) near them.

This talk is based on joint work with Renato Huzak and Maja Resman.

References

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