Polyhedral Neighboroods vs Tubular Neighboroods :

New Insights for the Fractal Zeta Functions

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Fractal zeta functions are classically often obtained by means of tube formulas. However, when it comes to fractals obtained by means of means of nonlinear and noncontractive iterated function system (i.f.s.), as is the case for the Weierstrass Curve, difficulties arise. Indeed, we can obtain the expression of the volume of the tubular neigborhood of the m^{th} prefractal approximation – and, hence, of the associated geometric tube zeta function, and the exact values of the poles ; i.e., of the Complex Dimensions associated to each prefractal approximation. However, a natural question is wether the Complex Dimensions of the fractal are the same as those of the prefractal approximations.

If we consider the fractal zeta functions associated with the sequence of polyhedral neighborhoods, instead of the fractal zeta functions associated with the sequence of tubular neighborhoods, not only do we dispose of the exact expression of the polyhedral neighborhood, at a given step of the prefractal approximation, but we also have the recurrence relation between consecutive fractal zeta functions. This, in particular, enables us to prove that the limit fractal zeta function – the one associated with the limit fractal object – has the same poles as the fractal zeta function at a given step of the prefractal approximation, and, hence, that the Complex Dimensions of the fractal are the same as the Complex Dimensions of a given prefractal approximation.