RATIONAL SOLUTIONS AND RATIONAL LIMIT CYCLES OF ABEL DIFFERENTIAL EQUATIONS

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Consider the Abel differential equation

$$x' = A(t)x^3 + B(t)x^2, \tag{0.1}$$

where A, B are polynomials or trigonometric polynomials. In this talk we consider two related problems.

First, we study the rational solutions when A, B are complex polynomials. We prove that if $\deg(A)$ is even or $\deg(B) > (\deg(A)-1)/2$ then the equation has at most two rational solutions. For any other case, an upper bound on the number of rational solutions is obtained. Moreover, we prove that if there are more than $(\deg(A) + 1)/2$ rational solutions then the equation admits a Darboux first integral.

Secondly, we study the number of rational limit cycles when A, B are trigonometric polynomials. We prove that this number is at most the degree of $A(t)$ plus one.

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